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Long wave asymptote for the Landau–Pomeranchuk–Migdal effect

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Abstract
It is shown that non-small angle multiple elastic scattering in matter leads to much stronger suppression of bremsstrahlung (BS) by high-energy particles in the long wave range of emission spectrum compared with the quenching predicted by Landau ŁD and Pomeranchuk I Ya (1953 *Dokl. Akad. Nauk SSSR* **92** 535, 735) and Migdal A B (1954 *Dokl. Akad. Nauk SSSR* **96** 49, 1956 *Phys. Rev.* **103** 1811). This manifests itself as the rearrangement of the BS spectrum of soft photons in the far long wave region.

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1. Introduction

Soft photons are a very important source of information on the properties of matter. Since the time of production of such particles is rather large they manage to 'test' the matter due to interaction therein before they escape it. In this way, the more soft photons, the more detailed information about the matter they carry.

The influence of scattering in matter on bremsstrahlung (BS) by high energy particles was studied by Landau and Pomeranchuk [1, 2] for the first time. The suppression of the intensity of the BS due to multiple elastic collisions of ultrarelativistic particles in matter (the Landau–Pomeranchuk effect) was pointed out in those papers. The quantitative theory of this effect has been derived by Migdal (the LPM effect) in [3, 4]. The theory of the LPM effect has been developed further in the study of the influence of the dispersion properties of scattering matter [5, 6], its boundaries [7, 8] and the Coulomb scattering of particles in matter [9] on the production of soft photons. In [10, 11] the BS in matter has been studied by means of the method of continual integrals[1]. In this way the principal approximation used in the papers mentioned above is the small angle scattering of particles in matter. As will be shown below

[1] A detailed review of publications devoted to the LPM effect is given in [12].

such an approach leads to the restriction to the application of the results obtained in [1–11] in the far long wave region.

In the present paper we study the production of soft photons by high-energy particles ($E \gg \omega$, where E and ω are the energy of the particle and the photon, respectively) which undergo multiple elastic collisions in amorphous infinite matter. It is shown that the BS in the far long wave range of the spectrum is formed due to the crucially non-small angle scattering of particles in the medium. This leads to dramatic differences of the spectral distribution of the BS of soft photons compared with that calculated in [1–4].

The paper is organized as follows. In section 2 we present intuitive estimations of the effect. In section 3 we study the soft BS by high-energy particles undergoing multiple elastic collisions in matter. In section 4 the influence of the inelastic scattering of particles in a medium on the emission spectrum is considered. Finally, in section 5 we present a brief summary and concluding remarks.

2. Intuitive estimation

The main approximation used in [1–11] cited above is the small angle elastic scattering of particles in matter. This also concerns the mean square of the multiple scattering angle of the particle during the time of formation of the long wave photon under the LPM effect. Let us calculate it.

Let us consider an ultrarelativistic particle undergoing multiple elastic collisions in infinite homogeneous matter. We assume the mean square of the multiple scattering angle of the particle per unit path length is q, but the photon energy is ω. Then, the time τ of photon generation in the LPM effect is given by the formula [6, 12]

$$\tau \sim 1/\sqrt{qv\omega}$$

where v is the particle velocity ($\hbar = c = 1$).

Then, the mean square ϑ_τ^2 of the angle of multiple scattering of the particle under the LPM effect (which should be small) is

$$\vartheta_\tau^2 = qv\tau \simeq qv\frac{1}{\sqrt{qv\omega}} = \sqrt{\frac{qv}{\omega}} \ll 1.$$

This means that the suppression of the BS in the form derived in [1–4] only takes place up to frequencies $\omega \gg vq$ because of the approximation of the small angle scattering of particles in matter which has been used therein.

In this way, we should note that, as well as multiple scattering, other mechanisms of suppression of the soft BS in matter take place. They are pair creation and dispersion of the medium [12].

The influence of these processes is significant when the length of photon production with respect to them is less than the length L_{ms} of photon emission due to multiple scattering in the matter.

In particular, in electrodynamics the length of pair creation L_{pc} is of the order of X_0, where X_0 is the radiation length [12]: $L_{\mathrm{pc}} \sim X_0$. In the case of non-small angle of scattering L_{ms} can be estimated as $L_{\mathrm{ms}} \sim q^{-1}$. Then, the influence of the pair production on the photon rate is strong when

$$X_0 \leqslant q^{-1} \sim X_0 \left(\frac{E}{E_s}\right)^2.$$

It follows from the last formula that pair creation is significant for rather fast electrons whose energy is much greater than $E_s = m\sqrt{4\pi/\alpha} = 21.2\,\mathrm{MeV}$ [12].

As for the influence of the dispersion of a medium on the photon production in matter, it strongly depends on the polarization properties of the scattering matter.

3. Bremsstrahlung by high-energy particles in matter in the long wave region of the spectrum

Let us consider an ultrarelativistic particle undergoing multiple elastic collisions in infinite scattering matter. We assume that the energy and mass of the particle are E and m, respectively. According to [1–4] the spectral distribution of the emission energy of a particle when its energy E is rather large, $E \gg \omega$, is given by the following expression:

$$\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega} = 2\,\mathrm{Re}\,\frac{\alpha\omega^2}{4\pi^2}\int_{-\infty}^{\infty}\mathrm{d}t\int_0^{\infty}\mathrm{d}\tau\,\exp(-\mathrm{i}\omega\tau)\int\mathrm{d}\Omega_1\int\mathrm{d}\Omega_2\int\mathrm{d}\Omega_{\vec{n}} \\ \times F_1(\Omega_1,t)F_2(\Omega_1;\Omega_2,\vec{k},\tau)(\vec{n}\times\vec{v}_1)(\vec{n}\times\vec{v}_2) \qquad (1)$$

where $\vec{v}_1$ and $\vec{v}_2$ are the velocities of the particle at moments t and $t+\tau$; ω is the emission frequency (the photon energy); $\vec{k} = \omega\vec{n}$ is the wave vector of a photon; Ω_1, Ω_2, $\Omega_{\vec{n}}$ are the solid angles along the directions of vectors $\vec{v}_1$; $\vec{v}_2$; $\vec{n}$, respectively, α is the fine structure constant and $\hbar = c = 1$.

The distribution functions F_1 and F_2 satisfy [3, 4] the standard kinetic equation.

3.1. Diffusion approximation

Let us consider the situation when the angle of a single scattering of particles is much smaller than the angle of their multiple scattering in the matter. We also assume that the distribution function is a rather smooth function of its variables. Such a situation is known as the diffusion approximation [13, 14]. Then, the function F_2 satisfies the following equation:

$$\frac{\partial F_2(\Omega_1;\Omega_2,\vec{k},\tau)}{\partial\tau} - \mathrm{i}\vec{k}\vec{v}_2F_2(\Omega_1;\Omega_2,\vec{k},\tau) = \frac{qv}{4}\triangle_{\vartheta_2,\varphi_2}F_2(\Omega_1;\Omega_2,\vec{k},\tau) \qquad (2)$$

$$F_2(\Omega_1;\Omega_2,\vec{k},\tau=0) = \frac{1}{2\pi}\delta(\cos\vartheta_1 - \cos\vartheta_2) \qquad (3)$$

where $\triangle_{\vartheta_2,\varphi_2}$ is the angle part of the Laplacian; ϑ_2, φ_2 are the polar and azimuth angles of the velocity $\vec{v}_2$ and q is the mean square of the angle of multiple scattering of a particle per unit path length[2].

Let us measure all angles from the direction of the vector $\vec{k}$. Then, it is convenient to introduce the new variables z and s and the new function f_2 according to the formulae

$$z^2 = \sqrt{\frac{\omega}{qv}}\sin^2(\vartheta_2/2) \qquad s = \tau\cdot\sqrt{vq\omega} \qquad (4)$$

$$F_2 = f_2\exp(\mathrm{i}kv\tau). \qquad (5)$$

Taking into account equations (2), (4) and (5) we derive the following equation for the function f_2:

$$\frac{\partial f_2(z,s)}{\partial s} + 2\mathrm{i}vz^2 = \frac{1}{16}\frac{\partial}{\partial z}\left[z\left(1 - z^2\sqrt{\frac{qv}{\omega}}\right)\right]\frac{\partial f_2(z,s)}{\partial z}. \qquad (6)$$

[2] The equation for the function F_1 can be obtained setting $\vec{k} = 0$ in equation (2).

The term on the right-hand side of equation (6) which corresponds to the azimuth part of Laplacian is omitted because of the axial symmetry of the considered problem.

In the case of the scattering of the particle through small angles, when $\vartheta_2 \ll 1$, it follows from equations (4) that $z^2\sqrt{qv/\omega} \ll 1$. Then, ignoring the term containing the frequency ω on the right-hand side of equation (6), we deal with the equation for the function f_2 in the small angle approximation [3, 4]. After the solution of such an equation and the substitution of the obtained function f_2 into formula (1) we come to Migdal's spectral distribution [3, 4] of the BS by an ultrarelativistic particle (v_1). The long wave asymptote $qv \ll \omega \ll qv(1-v)^{-2}$ of the emission energy found in [3, 4] takes the form (the LPM effect):

$$\frac{\mathrm{d}E_\omega}{2T\,\mathrm{d}\omega} = \frac{\alpha v^2(qv\omega)^{1/2}}{\pi} \tag{7}$$

where $\mathrm{d}E_\omega/2T\,\mathrm{d}\omega$ is the energy emitted in unit time, and T is the observation time.

As has been shown above (see section 2 and the text before equation (7)) the spectral distribution (7) is not applicable in the range of rather small frequencies $\omega \ll qv$ due to the approach of small angle scattering of particles in matter which has been used in [3, 4].

Let us consider the situation when $\omega \ll qv$, but the angles of multiple scattering are not small. In this case the parameter z in equation (4) is small: $z \ll 1$. Then, omitting the last term on the left-hand side of equation (6) we find the function $F_2(\Omega_1; \Omega_2, \vec{k}, \tau)$ satisfying equations (2) and (3) at $\omega \ll qv$:

$$F_2(\Omega_1; \Omega_2, \vec{k}, \tau) = \frac{1}{2\pi}\sum_0^\infty \left(\frac{2l+1}{2}\right) P_l(\cos\vartheta_1) P_l(\vartheta_2) \exp\left(-\frac{qv}{4} l(l+1)\tau\right). \tag{8}$$

The function F_1 appearing in equation (1) can be obtained from equation (8) by means of the substitutions $\cos\vartheta_2 \to \cos\vartheta_1$ and $\cos\vartheta_1 \to \cos\vartheta_k$, where ϑ_k is the emission angle.

Inserting the obtained F_1 and F_2 in formula (1), we find

$$\frac{\mathrm{d}E_\omega}{2T\,\mathrm{d}\omega} = \frac{4\alpha v\omega^2}{3\pi q}. \tag{9}$$

It follows from the spectral distribution (9) that dramatically stronger suppression ($\mathrm{d}E_\omega/2T\,\mathrm{d}\omega \propto \omega^2$) of the BS in the far long wave range of the spectrum takes place compared with the quenching ($\mathrm{d}E_\omega/2T\,\mathrm{d}\omega \propto \omega^{1/2}$) which has been predicted in [1–4] earlier. This is connected directly with the fact that the angles of the multiple elastic scattering of the particle in the matter are not small.

3.2. Long wave bremsstrahlung in matter beyond the diffusion approximation

Let us consider the emission by the high-energy particle when the diffusion approximation with respect to the elastic collisions of the particle in matter does not hold, i.e. we study the situation when the angles of both single and multiple scattering may have values of the same order.

In this case the function $F_2(\Omega_1; \Omega_2, \vec{k}, \tau)$ (see equation (1)) satisfies the Boltzmann equation in the general form instead of equation (2):

$$\frac{\partial F_2(\cos\theta_1, \cos\theta_2, \tau)}{\partial\tau} - \mathrm{i}\vec{k}\vec{v}_2 F_2(\cos\theta_1, \cos\theta_2, \tau) = \nu\left\{\int_{-1}^{1} \mathrm{d}(\cos\beta)\chi(\cos(\theta_2-\beta)) \cdot F_2(\cos\beta, \cos\theta_1, \tau) - F_2(\cos\theta_1, \cos\theta_2, \tau)\right\} \tag{10}$$

where ν is the collision frequency of the particle in the matter and θ_1 and θ_2 are the polar angles of the vectors $\vec{v}_1$ and $\vec{v}_2$. As above, all angles in equation (10) are measured from the direction of the vector $\vec{k}$.

Both the collision frequency ν and function $\chi(\cos\theta)$ are determined by the individual pair collisions of particles in matter. The collision frequency is connected with the total cross section σ of the elastic scattering of particles in matter by means of the standard expression,

$$\nu = n \cdot v \cdot \sigma \tag{11}$$

while the function χ depends on both differential and total cross sections

$$\chi(\cos(\theta)) = \frac{2\pi\, \mathrm{d}\sigma(\cos(\theta))}{\sigma\, \mathrm{d}\Omega} \tag{12}$$

where n is the density of scattering centres in the medium, v is the particle velocity and Ω is the solid angle.

In case of the emission of rather soft photons $\omega \ll \nu$, we can neglect the left-hand side of equation (10) which contains the vector $\vec{k}$. This is possible since at $\omega \ll \nu$ the scattering of the particle in matter is not small. Then, expanding F_2 and χ in equation (10) in the complete set of Legendre polynomials $P_l(\mu)$, we obtain

$$F_2(\vartheta_1, \vartheta_2, \tau) = \sum_{l=0}^{\infty} \frac{2l+1}{2} P_l(\cos\vartheta_1) P_l(\cos\vartheta_2) \exp\left(-\nu\tau + \frac{2\nu}{2l+1} \cdot \chi_l \cdot \tau\right) \tag{13}$$

where χ_l is given by the formula

$$\chi_l = \frac{2l+1}{2} \int_{-1}^{1} \chi(\mu) P_l(\mu). \tag{14}$$

The function F_1 appearing in equation (1) can be derived from equation (13) by means of the substitutions $\cos\vartheta_2 \to \cos\vartheta_1$ and $\cos\vartheta_1 \to \cos\vartheta_k$, where ϑ_k is the emission angle.

Substituting F_1 and F_2 given by equation (13) into formula (1) we get the spectral distribution of the emission energy in the long wave region:

$$\frac{\mathrm{d}E_\omega}{2T\, \mathrm{d}\omega} = \frac{2\alpha v^2 \omega^2}{3\pi\nu\,(1 - 2\chi_1/3)} \qquad \omega \ll \nu. \tag{15}$$

It follows from the spectral distribution (15) that, as has been stated above (see equation (9)), strong suppression of the BS by high-energy particles in the far long wave range of the emission spectrum takes place. The quenching is significantly stronger ($\mathrm{d}E_\omega/2T\, \mathrm{d}\omega \propto \omega^2$) than has been predicted in [1–4] where ($\mathrm{d}E_\omega/2T\, \mathrm{d}\omega \propto \omega^{1/2}$). Such suppression, as it takes place in the case of diffusion approximation (see the text after equation (9)), is connected directly with the non-small angles of scattering of the particle in matter.

Note that the dependence of the emission energy on the photon energy is like that in the diffusion approximation, while it only differs by the normalization factor.

4. Influence of inelastic scattering on the formation of the long wave asymptote of bremsstrahlung in matter

In the diffusion approach the influence of inelastic collisions on the multiple elastic scattering of particles in matter is determined by the relation between two parameters. They are, so called, the transport length $l_{\mathrm{tr}} \simeq q^{-1}$ and the loss length l_{loss}. In the case of collisions of electrons in solids we have the following [14] for these characteristics:

$$\frac{l_{\mathrm{loss}}}{l_{\mathrm{tr}}} \simeq \frac{m(Z+1)L_{\mathrm{C}}}{EL_{\mathrm{i}}} \tag{16}$$

where Z is the charge of the nucleus and L_C and L_i are the Coulomb and ionization logarithms, respectively. As a rule, we can approximately set $L_C \sim L_i$. Then, in the case of rather heavy materials, so that $Z \gg E/m$, we have $l_{tr}/l_{loss} \ll 1$ even for ultra-relativistic electrons (when $E/m \geqslant 10$, for example). This means that there are some realistic situations when neglecting the influence of the inelastic scattering on the production of the soft BS photons in matter is absolutely correct. In such cases the BS spectrum is given by equation (9).

The situation becomes more complicated when the diffusion approximation is invalid. In this case the distribution function F_2 appearing in equation (1) satisfies the Boltzmann equation in the following form [14]:

$$\frac{\partial F_2(\cos\theta_1, \cos\theta_2, \tau)}{\partial \tau} - \mathrm{i}\vec{k}\vec{v}_2 F_2(\cos\theta_1, \cos\theta_2, \tau)$$
$$= \nu \left\{ \int_{-1}^{1} \mathrm{d}(\cos\beta)\chi(\cos(\theta_2 - \beta)) \cdot F_2(\cos\beta, \cos\theta_1, \tau) - F_2(\cos\theta_1, \cos\theta_2, \tau) \right\}$$
$$+ \frac{1}{E} \left\{ \int_0^{\infty} \frac{\mathrm{d}\epsilon}{E} w(E+\epsilon) F_2(\cos\beta, \cos\theta_1, \tau) - w(E) F_2(\cos\theta_1, \cos\theta_2, \tau) \right\} \tag{17}$$

where $w(E)$ is the energy lost by the particle per unit time and E is the particle energy.

In the limit of the long wave photon $\omega \ll \min(\nu; w(E)/E)$ we can omit the term on the left-hand side in equation (17) which contains the photon momentum $\vec{k}$. Although the solution of equation (17) cannot be found in the explicit form it nevertheless does not depend on the photon energy at $\omega \ll \min(\nu; w(E)/E)$. This means that after the substitution of F_2, satisfying equation (17), into formula (1) we go to the spectrum

$$\frac{\mathrm{d}E_\omega}{2T\,\mathrm{d}\omega} = \frac{2\alpha v^2 \omega^2}{3\pi \nu_{tot}} \qquad \omega \ll \nu_{tot} \tag{18}$$

where ν_{tot} designates some frequency which characterizes the collision of the particle in matter. It takes into account both elastic and inelastic impacts in the medium and does not depend on the photon energy at $E \gg \omega$.

It follows from equation (18) that the inelastic collisions only lead to the renormalization of the emission rate but they do not change the dependence of the emission energy on radiation frequency. It is obvious that the analogous renormalization also takes place in the diffusion approximation at $Z \leqslant (E/m)$.

We should note that even if the representation of the collision integral as the sum of the elastic and inelastic parts (see equation (17)) is impossible the dependence of the emission energy on the radiation frequency in the long wave range of the spectrum remains in force and is given by equation (18). This is related to the fact that in such a region of the spectrum we can ignore the term containing the photon momentum on the left-hand side of the kinetic equation.

Thus, the dependence $\mathrm{d}E_\omega/2T\,\mathrm{d}\omega \propto \omega^2$ of the long wave BS in matter on photon energy which is given by equation (18) is very general. It dramatically differs from the dependence of the emission energy on radiation frequency in the case of the individual (non-coherent) collisions of particles where $\mathrm{d}E_\omega/2T\,\mathrm{d}\omega$ does not depend on ω [15] at all. Such behaviour of $\mathrm{d}E_\omega/2T\,\mathrm{d}\omega$ as a function of ω is directly related to the presence of the multiple scattering of particles in matter.

5. Conclusion

The far long wave BS by particles undergoing multiple scattering in matter is studied. It is shown that the extremely soft BS is formed due to the crucially non-small angle of scattering of particles in the medium. The spectral distribution of the BS of soft photons by such particles is obtained. The calculated spectrum dramatically differs from that obtained before in [1–4]. The differences manifest themself in the dependence of the emission energy on the parameters of the problem studied (emission frequency and characteristics of scattering matter). In particular, much more suppression of the soft BS takes place compared with the quenching predicted in [1–4].

The influence of inelastic collisions on the BS spectrum in matter is studied. It is shown that the inelastic scattering renormalizes the value of the intensity of the BS but does not change the dependence of the emission energy on the radiation frequency compared with the case when only elastic collisions take place.

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